

Example of Method Used in “Ranking Doctoral Programs by Placement: A New Method”

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An example illustrates how this method operates in a small closed system, with $q = 0$. Assume there are only three universities—A, B, and C—which grant all Ph.D.s in the field and hire (from the same three) all faculty members in the field. Over a ten-year period, A hires eight new tenure-track professors, B hires five, and C hires four. In that same period, each program awards 10 Ph.D.s, of which A successfully places 9 in faculty positions and B and C each place 5. A places 5 at A, 2 at B, and 2 at C. B places 2 at A, 1 at B, and 1 at C. C places 1 at A, 2 at B, and 1 at C. These data can be represented by the following matrix:

$$\begin{bmatrix} 5 & 2 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

To calculate our weighted influence measure from these data, we would first divide each row by 10 (the number of Ph.D.s awarded by each institution over this period). In this case, the weighted and unweighted measures produce the same result because the Ph.D. programs are of equal size. This can be seen in the following step, as when the columns are normalized such that they sum to 1 the result is the same regardless of whether the initial matrix is first divided by 10. The resulting stochastic matrix is:

$$\begin{bmatrix} 0.625 & 0.4 & 0.5 \\ 0.25 & 0.2 & 0.25 \\ 0.125 & 0.4 & 0.25 \end{bmatrix}$$

As stated earlier, this method of ranking can be thought of as a series of rounds of voting in

which universities “vote” for their peers and themselves by hiring graduates of those programs as their new tenure-track faculty members. Each university initially gets one vote, which is divided among the graduate programs it hires its faculty from. For example, in the above example A has given 0.625 of its vote to itself, 0.25 of its vote to B, and 0.125 of its vote to C. In the first “round” of voting, a university’s total number of “votes” received is simply the sum of its row. This can also be represented by the following matrix-vector multiplication (all values are rounded to three decimal places):

$$\begin{bmatrix} 0.625 & 0.400 & 0.500 \\ 0.250 & 0.200 & 0.250 \\ 0.125 & 0.400 & 0.250 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.525 \\ 0.700 \\ 0.775 \end{bmatrix}$$

In this first round, A comes out on top because it has an overall placement rate (90%) that is greater than that of B and C. Although B and C have the same overall placement rate (50%) and number of graduates (5), C does better than B in the first round because it placed its graduates at departments with fewer hires on average than B did (and a placement at a smaller department counts more, at least in the first round of voting, because it is a larger percentage of the university’s total hiring).¹ Consequently C garnered a larger total vote share than B. In the second round of voting, each university’s voting strength corresponds to the total number of votes it received in the first round. Thus A’s vote counts more heavily than C’s, and C’s counts more than B’s. The second-round matrix-vector multiplication is:

$$\begin{bmatrix} 0.625 & 0.400 & 0.500 \\ 0.250 & 0.200 & 0.250 \\ 0.125 & 0.400 & 0.250 \end{bmatrix} * \begin{bmatrix} 1.525 \\ 0.700 \\ 0.775 \end{bmatrix} = \begin{bmatrix} 1.621 \\ 0.715 \\ 0.664 \end{bmatrix}$$

A now takes a commanding lead (as we would expect, given its greater overall placement rate), and B passes C to take second place. Although C initially beat B for the reasons described above, in this round the greater weight (1.525) given to A’s initial vote (0.25 for B and 0.125 for C) helped B more than it helped C, the smaller weight (0.7) given to B’s

¹The assumption that the three programs award the same number of Ph.D.s, thereby making the weighted and unweighted measures equivalent, allows us to discuss the role of placement *rates* in the rankings. If this were not the case, the placement rate interpretation would only make sense for the weighted measure. For the unweighted measure, the total votes in the first round would be a function of the raw number (not percent) of graduates placed in faculty positions and the size (number of faculty) of the departments hiring these graduates (a hire by a department with a small number of faculty counts for more than a hire by a larger department).

initial vote (0.2 for itself and 0.4 for C) hurt C more than it hurt B, and the weight (0.775) given to C's initial vote (0.25 for B and 0.25 for itself) had the same effect on B and C. A third round of voting yields the following ranking (total votes) vector:

$$\begin{bmatrix} 1.631 \\ 0.714 \\ 0.655 \end{bmatrix}$$

The third-round vector is much more similar to the second-round vector than the second-round vector is to the first-round vector. In fact, it is very close to the vector that would be obtained after an infinite number of rounds of voting (also called the dominant eigenvector of the matrix). The above (third-round) vector can be scaled such that the highest ranked university has a score of 100, which results in the following ranking vector:

$$\begin{bmatrix} 100.000 \\ 43.790 \\ 40.137 \end{bmatrix}$$

The actual dominant eigenvector of the matrix (calculated using a computer) is a vector that can be scaled to the following:

$$\begin{bmatrix} 100 \\ 43.75 \\ 40 \end{bmatrix}$$

An application of this methodology using real data (and all Ph.D.-granting institutions in the United States) takes longer to calculate, but this simplified example illustrates some of its key features. At the base of this methodology is an advantage that is awarded to graduate programs with high overall placement rates, as the above example shows. However, it goes beyond a simple ranking based on overall placement rates to take into account the placement rates of the programs at the hiring institutions where the program has placed its graduates (and the placement rates of the programs at the institutions that hired graduates from the hiring institutions just mentioned, etc.). The above example showed this: B and C had the same overall placement rate, but in the end B “beat” C because B placed more of its graduates at A (where the graduate program has a higher overall placement rate than B and C) than did C.